Physics 122
Chapter 17
Problem Solutions

Q1 If you find yourself confused by questions that involve electrical potential and potential energy you can try making an analogy with gravitational potential energy near the surface of the Earth. In this case we can define the gravitational field as $\vec{g} = 9.8m/s^2$ pointing downward. This is analogous to the electric field. The gravitational force is $m\vec{g}$ which is analogous to the electrical force $q\vec{E}$[*]. Finally the gravitational potential energy, $PE = mgh$, will be analogous to an electrical potential energy, $PE = qED$. Here it is assumed that positive h is upward (the opposite direction to $\vec{g}$) and so positive D is in the opposite direction as E.

Think of potential energy as the potential energy of a small ball on a lumpy surface. The value of the potential energy is proportional to the altitude of the ball. If you have the ball in at an elevation of 10cm and then it gets moved to a different place that also has an elevation of 10cm, did you do any net work to move it?

Suppose that the initial and final kinetic energies are the same. Then the work energy theorem says that the work done is the sum of the change in kinetic energy (that would be zero) and the change in the potential energy (also zero). No work would have been done. Notice that this result depends entirely upon the assumption that the kinetic energy remained constant. The problem does not state explicitly that this is true but I think that it is pretty clear that this is what the author intended.

Can you necessarily get from one point to the other without the application of a force if the two points are at the same potential energy? Again, think of the analogy. If you have two points at the same elevation but they are in different valleys separated by a ridge you will have you use a force to push the ball up the hill on one side but then you get that work back as you go down into the other valley. The net work will be zero but force is required to make the process happen. This is a very common concept in chemistry when reaction kinetics are being analyzed.

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[*] Notice that the mass is always positive while the charge can be positive or negative. Thus the gravitational force is always parallel to $\vec{g}$ but the electrical force might be anti-parallel to $\vec{E}$.

Q2 This question points out an essential difference between the electric potential and the electric potential energy. These ideas tend to be confusing because your earlier experience is with gravitational potential energies which depend upon mass. Unlike mass, charge has two flavors – positive and negative. So while rocks always fall down, in an analogous situation charged objects sometimes go down (in the same direction as the electric field – these are positive charges) and sometimes go up (in the opposite direction as the electric field – these are negative charges.)

With that said, the charged object at rest will move in the direction of the force that acts upon it (Newton's 2nd law). The definition of the electric field can be rewritten as

$$\vec{F} = q\vec{E} \tag{Q2-1}$$

If we move in a straight line by some displacement $\vec{D}$ then the work done by the electric force in (Q2-1) will be

$$W = q\vec{E}\bullet\vec{D} \tag{Q2-2}$$

and the corresponding change in the electrical potential energy will be the negative of this expression.[†]

$$\Delta PE = -q\vec{E}\bullet\vec{D}\ ^{\ddagger} \tag{Q2-3}$$

Suppose that the charge q is positive. Then the electrical force does push us in the direction of the electric field. If we move in the direction we are pushed then $\vec{D}$ is in the same direction as $\vec{E}$ and $\vec{E}\bullet\vec{D} = ED$. The work term will be positive and the change in potential energy will be negative. The charge "falls" to the lower potential energy.

Suppose though, that the charge q is negative. The force will push in the opposite direction as the field is pointing. Thus, if we move in the direction that the force pushes us, $\vec{E}\bullet\vec{D} = -ED$. However, q is also negative so that the work term, (Q2-2) is again positive and we have the same result as before; the charge "falls" to the lower potential energy. Note however that this location is different. For the positive charge we moved in the same direction as the electric field and for the negative charge we moved in the opposite direction.

Both types of charge will be pushed by the field toward a state of lower potential energy. What about electric potential though? Electric potential differences are

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[†] Recall that potential energy differences are just the negative of the corresponding work terms. The expression just gets moved from the left hand side of the work-energy theorem to the right hand side of the same.

[‡] Note that this is consistent with the 1st paragraph of Q1. Be sure that you see how this is so.

simply the electric potential energy differences divided by the charge of the object. In both cases (positive charge and negative charge) the potential energy difference was positive. Now, however, we divide once by +q and the other time by –q.

- The positive charge moves to a lower potential energy and also to a lower electrical potential.
- The negative charge moves to a lower potential energy but, at the same time, to a higher electrical potential.

When thinking about electrical potential one could say that positive charges naturally roll "downhill" while negative charges naturally roll "uphill."

Q8 In the previous two problems we looked at the relationship of electrical potential to work and energy. In this problem we are concerned about its relationship to the electric field.

Because the electric potential is a potential – just like potential energy – we are only interested in how it changes as we move from one place to another. Thus, we are free to set the value of the electrical potential at any one point in space[§] to zero or any other value we choose. Nothing need be said about the electric field to make this assignment.

For the first question simply select a point in space where the electric field is not zero and define the electrical potential to be zero there. For the second question, select a point in space where the electric field is zero and define the electrical potential to be something other than zero.

This might seem a bit arbitrary and silly so let's look carefully at the actual relationship between the two quantities and then make an analogy to help us understand it.

We begin with the electrical potential and then ask for the electric field. Suppose that in some region of space you have selected a particular point to be the zero of electrical potential. Other points have values that vary from zero. How can I find the value of the field at a particular point? Suppose that the potential at my point is V_0. I look a very short distance away in all directions. In each direction the potential will have some other value and one of them will differ from V_0 more than any of the others; I pick this as a second point in space. The electric field will point in the direction from the larger of these two potentials to the smaller. The size of the field will be the difference between the two potentials divided by the distance between them.[**]

Notice that neither of these two quantities (direction or size) would change if I added some constant number to the potential at all of the points in this region of

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[§] Except for places exactly coincident with point charges.

[**] The orange text is here as a marker for Q11.

space. This is why I can choose any point I wish to be the zero of electrical potential.

An analogous relationship can be found in position and velocity. Suppose that there is an object moving around in space. You could pick the location at some instant in time and ask for the velocity. How would you find it? You could look at the position a short time later. The velocity vector would point from the initial location to the final location and its size would be the distance between the two divided by the elapsed time. If you had shifted the origin of your coordinate system, all of the positions would change by the same amount but when you calculated the velocity that shift would get subtracted away.

The analogy, then, is that the potential is like the position in space and the electric field is like the velocity. The electric field, like the velocity, has a definite value at any given point. The potential, like the position, depends upon where you put your origin.

Finally, let's give concrete examples of two cases. One will have a point in space where, with conventional choices for the zero of potential, $V = 0$ and $\vec{E} \neq 0$. The other will have $\vec{E} = 0$ and $V \neq 0$.

1st Example.

Place a charge of +Q on the x axis at a position of +L. Place a second charge this one –Q at –L on the x axis. Now consider the origin with the conventional choice of V=0 at an infinite distance from the charges. The origin is an equal distance from each charge and so the total potential at this point is simply the sum of the potential due to each charge by itself.

$$\begin{aligned} V_{Total} &= V_{-} + V_{+} \\ &= \frac{k \cdot (-Q)}{L} + \frac{k \cdot (+Q)}{L} \qquad \text{(Q8-1)} \\ &= 0 \end{aligned}$$

But what is the electric field at this point? We can calculate it using Coulomb's law.

$$\vec{E}_{Total} = \vec{E}_- + \vec{E}_+$$

$$= \left[\frac{k \cdot (-Q) \cdot \hat{x}}{L^2}\right] + \left[\frac{k \cdot (+Q) \cdot (-\hat{x})}{L^2}\right]$$

$$\frac{-2k \cdot Q \cdot \hat{x}}{L^2} \tag{Q8-2}$$

$$\neq 0$$

2nd Example

Set up the charges exactly as in example 1 but make them both positive. Repeat the computation in (Q8-1) and (Q8-2). What do you get?

Q11 This question is closely related to Q8. We are asked for the electric field in a region of space that has the electric potential behaving is a specified manner. Go back and read the parts of Q8 that are printed in orange. Can you answer the question now?

We are given that the electric potential does not vary from point to point in this region of space. What does this imply for the electric field? How do you calculate the field if you have knowledge of the potential? Try to answer the question again.

The size of the field is the ratio of the change in the electric potential between two nearby points to the distance between those points. If the difference in the potential is zero then the size of the electric field is zero and it has no direction.

P4 Just as the electric field at some point in space can be thought of as the force per unit charge at that location, the electric potential can be thought of as the potential energy per unit charge at that point.

$$\Delta V = \frac{\Delta PE}{Q} \tag{P4-1}$$

Here we are told that the electron gained $7.45 \cdot 10^{-16}$J of kinetic energy as it moved from one point to another. The reason that it did so was that there was an electric field pushing on it. This is like saying that I drop a rock and it gains kinetic energy as it moves from one altitude to another because the gravitational field was pushing on it.

In the absence of friction, the work energy theorem

$$W = \Delta KE + \Delta PE$$

tells us that the increase in the kinetic energy must be the same magnitude as the decrease in the potential energy. Thus $\Delta PE = 7.45 \cdot 10^{-16} J$. Looking up the charge on an electron we find that (P4-1) becomes

$$\Delta V = \frac{-7.45 \cdot 10^{-16} J}{-1.60 \cdot 10^{-19} C} \qquad \text{(P4-2)}$$

$$= 4656 J / C$$

$$= 4656 V$$

Because the change in the potential is positive (final minus initial) you can see that the electron moves from low potential to high potential. However, the final potential <u>energy</u> is less than the initial potential <u>energy</u>.

P5 The thing you have to know here is that the electric field between two parallel metal plates will be uniform. Think for a moment about uniform velocity. During the entire time interval for which the velocity is constant the ratio Δx/Δt is the same no matter what pair of points you might pick. Here we have ΔV/Δx that is uniform. Let's pick the points we know about – on the two plates.

The plates are $5.8 \cdot 10^{-3}$m apart and the potential difference is 220V. The electric field strength is then

$$E = \frac{220V}{5.8 \cdot 10^{-3} m}$$

$$= 3.79 \cdot 10^{4} V / m$$

P10 This problem statement is giving information about work and kinetic energy; it asks about electrical potential. This suggests that the work energy theorem would probably be useful.

$$W = \Delta KE + \Delta PE \qquad \text{(P10-1)}$$

Consider each part of (P10-1) in turn. The work is only that done by the external force as any electrostatic force will be accounted for by the potential energy term.

$$W = 15.0 \cdot 10^{-4}\,J \tag{P10-2}$$

The kinetic energy term is given to us as well.

$$\begin{aligned}\Delta KE &= \left(4.82 \cdot 10^{-4}\,J\right) - \left(0\,J\right) \\ &= 4.82 \cdot 10^{-4}\,J\end{aligned} \tag{P10-3}$$

All that remains is the potential energy term and it must contain the quantity we seek. Point a is the initial point and point b is the final point. I will say that the difference in potential between point a and point b is $\Delta V = V_b - V_a$. With this assignment I can write

$$\Delta PE = Q \cdot \Delta V \tag{P10-4}$$

Combine P10-1 through P10-4.

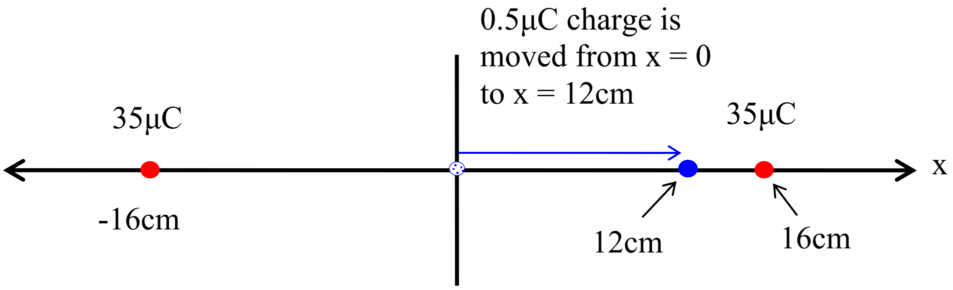
$$15.0 \cdot 10^{-4}\,J = 4.82 \cdot 10^{-4}\,J + \left(-8.50C\right) \cdot \Delta V$$

$$\Delta V = \frac{10.18 \cdot 10^{-4}\,J}{-8.50 \cdot 10^{-6}\,C} \tag{P10-5}$$

$$\Delta V = -120V$$

With my definition of ΔV, this means that the initial point, a, has a potential that is larger by 120 volts that that of point b.

P16 A picture will help us get started on this problem.



Because the electric field strength is changing as the small charge is moved we are not able[††] to calculate the work done from $W = \sum \vec{F} \bullet \Delta l$. However, we have been given the essential result of the calculation. The electric potential for a point charge Q is

$$V(r) = \frac{kQ}{r} \qquad \text{(P16-1)}$$

where r is the distance from the charge. Note that this is a scalar relation – there are no vectors present. It has also been assumed in this result that the zero of electrical potential is taken as locations far from the charge; $r \to \infty$. Because the principle of superposition holds for electric potential as well as the electric field, we can add the potentials due to the two fixed charges to get the overall potential that the small, 0.5μC charge sees.

$$V_{initial} = \frac{kQ_{left}}{r_{left}} + \frac{kQ_{right}}{r_{right}}$$

$$= \frac{9 \cdot 10^9 Nm^2/C^2 \cdot 35 \cdot 10^{-6} C}{0.16m} + \frac{9 \cdot 10^9 Nm^2/C^2 \cdot 35 \cdot 10^{-6} C}{0.16m} \qquad \text{(P16-2)}$$

$$= 3.94 \cdot 10^6 V$$

$$V_{final} = \frac{kQ_{left}}{r_{left}} + \frac{kQ_{right}}{r_{right}}$$

$$= \frac{9 \cdot 10^9 Nm^2/C^2 \cdot 35 \cdot 10^{-6} C}{0.28m} + \frac{9 \cdot 10^9 Nm^2/C^2 \cdot 35 \cdot 10^{-6} C}{0.04m} \qquad \text{(P16-3)}$$

$$= 9.00 \cdot 10^6 V$$

The implication of the problem statement is that the test charge begins and ends at zero speed so the change in kinetic energy is zero. By multiplying the change in the electrical potential by the size of the test charge we will be able to compute

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[††] It becomes a problem in integral calculus.

the change in the potential energy of the system and so use the work energy theorem to find the work done in moving the test charge.

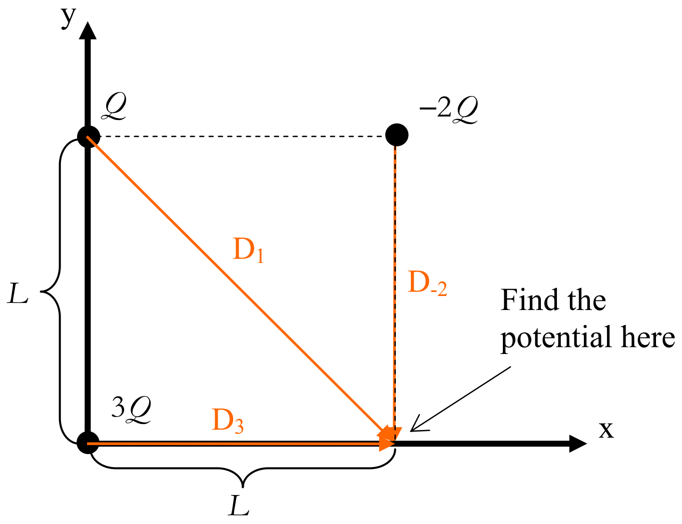
$$W = \Delta KE + \Delta PE \qquad \text{(P16-4)}$$

Fill into (P16-4) from (P16-2) and (P16-3).

$$W = 0 + \left[\left(5.0 \cdot 10^{-7} C\right)\left(9 \cdot 10^{6} V - 3.94 \cdot 10^{6} V\right)\right]$$

$$= 2.53 J$$

P19 This problem is closely related to P16. You should read over that solution and then try this one again if you are stuck.



The electric potential in the lower right corner of the square is the sum of the potentials due to the simultaneous presence of the three charges. If the zero of potential is, as usual, taken to be at points far away then we simply add together three scalar terms of the form

$$V(r) = \frac{kQ}{r} \qquad \text{(P19-1)}$$

I put a label on each of the displacement vectors that connect the charges to the field point. Notice that we only need their magnitudes. The potential is much easier to work with than electric field. Use (P19-1) three times.

$$V_{total} = \frac{k\cdot(3Q)}{D_3} + \frac{k\cdot(Q)}{D_1} + \frac{k\cdot(-2Q)}{D_{-2}}$$

$$= \frac{k\cdot(3Q)}{L} + \frac{k\cdot(Q)}{\sqrt{2}L} + \frac{k\cdot(-2Q)}{L}$$

$$= \frac{kQ}{L}\left[3 + \frac{1}{\sqrt{2}} - 2\right]$$

$$= 1.71\cdot\frac{kQ}{L}$$

P36 For this problem we need the definition of capacitance. In this case our two conductors are the capacitor plates. The ratio of the charge on the plates to the potential difference between them is a constant that we call the capacitance.

$$C = \frac{Q}{V} \qquad \text{(P36-1)}$$

What do we know? There will be two different voltages applied to the capacitor which will result in two different charges on the plates. Call these V_1 and V_2; Q_1 and Q_2. We are given the voltages but not the charges. Still, we can write these into (P36-1).

$$C = \frac{Q_1}{V_1} \qquad \text{(P36-2)}$$

$$C = \frac{Q_2}{V_2} \qquad \text{(P36-3)}$$

These last two equations contain three unknowns – the two charges and the capacitance. Thus we cannot solve them as they stand for the value of the capacitor. However, we have been told the difference between the two charges, ΔQ. Let us stipulate that $V_2 > V_1$. Then it follows that $Q_2 > Q_1$. That means then

$$\Delta Q = Q_2 - Q_1 \qquad \text{(P36-4)}$$

Now, with the addition of (P36-4) we have a system of three equations in three unknowns and you may use your preferred technique to solve them. **Stop reading right here and solve the system. This is the kind of algebra practice that many in this class need. If you are proficient at this task – good for you!**

If you find this difficult here is a chance to get better. Should you be stuck on this task, go ahead and follow through the lines below. But then, put them aside and work it out with your own hand. Reading and nodding will not help you to learn.

I want to find a value for C. That means I want to eliminate Q_1 and Q_2. If I solve (P36-2) for Q_1 in terms of C and V_1 and then get Q_2 in a similar way from (P36-3) I should be able to then use (P36-4) to make a single equation in the unknown C.

$$Q_1 = C \cdot V_1$$

$$Q_2 = C \cdot V_2$$

$$\Delta Q = C \cdot V_2 - C \cdot V_1$$

Now I need only solve this last one for C.

$$C = \frac{\Delta Q}{V_2 - V_1} \tag{P36-5}$$

$$C = \frac{18 \cdot 10^{-6} C}{121V - 97V}$$

$$= 7.5 \cdot 10^{-7} F$$

Here is something else to think about.

1. From the point of view of a graph of charge vs. voltage for a capacitor, what does (P36-1) tell you? It says that

$$Q = C \cdot V$$

2. What is this? It is an equation for a straight line that passes through the origin.

3. What is C? It is the slope of the line.

4. Is there another way you would express the slope of such a line? How about

$$C = \text{slope} = \frac{\Delta Q}{\Delta V}$$

Compare this to (P36-5)

P40 If you read through this problem you know that you are given the final voltage across capacitor #2 and asked for its size. From the definition of capacitance

$$C = \frac{Q}{V} \tag{P40-1}$$

you can see that only the charge is needed to produce our result.

Notice how this solution is starting out. I do not see exactly how the whole problem will be solved and I am surely not looking for some equation in my book that will give me the answer. What I do is to ask questions of my own. The first question is "What is capacitance?" and the answer is found in the form of a definition. This begets a second question "What would I need to know to use this definition to find my result?"

But what do we know about it?

Recall last semester when I always wanted you to have a movie of the problem that you could play in your head? Now I play the movie and watch the charge moving around.

We know that initially all of the charge was on capacitor #1 and then some of it moved over to capacitor #2. Let's write that down.

This may seem like a small thing but it is actually a huge hurdle for many people. I think that the largest part of it is simply realizing that you need to give names to things and then write out in a formal way the English sentence.

$$\begin{gathered} Q_{1,initial} = Q_{1,final} + Q_{2,final} \\ \text{or} \\ Q_{2,final} = Q_{1,initial} - Q_{1,final} \end{gathered} \tag{P40-2}$$

The second line tells me what I should be trying to do. Find values for the charges on capacitor #1 so that I can solve for the final charge on capacitor #2.

Is there some way of getting at the charge on capacitor #1? Sure! Just use (P40-1) and solve for the charge.

$$Q_{1,initial} = C_1 \cdot V_{1,initial} \tag{P40-3}$$

$$Q_{1,final} = C_1 \cdot V_{1,final} \tag{P40-4}$$

Now I can put (P40-3) and (P40-4) into (P40-2) to get the final charge on capacitor #2.

$$
\begin{aligned}
Q_{,final} &= C_1 \cdot V_{1,initial} - C_1 \cdot V_{1,final} \\
&= C_1 \cdot \left(V_{1,initial} - V_{1,final}\right)
\end{aligned}
\qquad \text{(P40-5)}
$$

And now (P40-1) allows me to find C$_2$. Recall that $V_{2,final} = V_{1,final}$.

$$
\begin{aligned}
C_2 &= \frac{Q_{2,final}}{V_{2,final}} \\
&= \frac{C_1 \cdot \left(V_{1,initial} - V_{1,final}\right)}{V_{1,final}} \\
&= C_1 \cdot \left(\frac{V_{1,initial}}{V_{1,final}} - 1\right) \\
&= 7.7\mu F\left(\frac{125V}{15V} - 1\right) \\
&= 56.5\mu F
\end{aligned}
$$

The solution process is the thing you should be studying. You are learning a way of thinking.

P50 In this problem we are given information about the geometry of a parallel plate capacitor and asked questions about the energy stored in the electric field. What kind of relationship do we need? We must be able to connect the amount of energy stored to the plate area, the plate separation, and the charge on the plates. To begin the process recall that the energy stored in a capacitor is

$$E = \frac{1}{2}QV \qquad \text{(P50-1)}$$

This can be rewritten with the definition of capacitance into two other forms.

$$E = \frac{1}{2}CV^2 \qquad \text{(P50-2)}$$

or

$$E = \frac{1}{2}\frac{Q^2}{C} \tag{P50-3}$$

Since there is no information given concerning the voltage, (P50-3) looks like the best starting point. Can C be related to the plate area and spacing? Yes, this was done for us in lecture and in the text book as well.

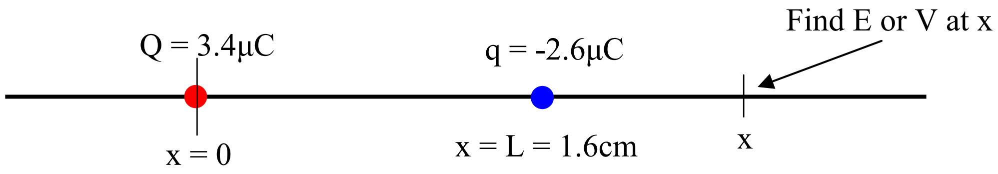
$$C = \varepsilon_0 \frac{A}{d} \tag{P50-4}$$

If we combine (P50-3) and (P50-4) we will have an expression for the energy stored in a capacitor as a function of the plate separation.

$$E = \frac{Q^2}{2A\varepsilon_0} \cdot d \tag{P50-5}$$

Now the answers to the questions are clear. If d is doubled while all else is held fixed then E doubles. The extra energy comes from the force that pulled the plates farther apart. The plates attract each other so the applied force will act in the same direction as the motion of the moving plate. The work done by this force is equal to the increase in the stored energy by the work energy theorem.

P63 This problem should remind you of Q20 in chapter 16. You might want to go review that problem if you are having trouble getting started. Notice the correspondence between these two problems. In particular note the relative magnitudes of the two charges in each problem.



Start with finding the place where the electric field is zero. The counterpart to equation Q20-1 from chapter 16 is

$$\vec{E} = \frac{kQ}{x^2}\hat{x} + \frac{kq}{(x-L)^2}\hat{x} \tag{P63-1}$$

This can only be zero if the coefficients of each of the two vectors on the right hand side are equal in size and opposite in sign.

$$\frac{kQ}{x^2} = -\frac{kq}{\left(x-L\right)^2} \tag{P63-2}$$

Solve this for x. I will leave you to fill in the missing steps here.

$$x = \frac{2LQ \pm \sqrt{4L^2Q^2 - 4QL^2\left(Q+q\right)}}{2\left(Q+q\right)} \tag{P63-3}$$

Note that $4QL^2\left(Q+q\right) = 4Q^2L^2\left(1+\frac{q}{Q}\right)$ so that (P63-3) can be simplified as

$$x = L\left(\frac{Q}{Q+q}\right)\left(1 \pm \sqrt{\frac{-q}{Q}}\right)$$

$$= 1.6cm\left(\frac{3.4\mu C}{3.4\mu C - 2.6\mu C}\right)\left(1 \pm \sqrt{\frac{2.6\mu C}{3.4\mu C}}\right) \tag{P63-4}$$

$$= \begin{cases} 12.7cm \\ or \\ 0.857cm \end{cases}$$

As was argued in Q20 in chapter 16 only a solution to the left of the negative charge can be correct. The electric field is zero at x = 12.7cm.

When we now look for the location of zero electrical potential[‡‡] we want to do much the same thing but our equation will be easier to solve. The potential of a positive charge will be positive and that due to a negative charge will be negative.

$$V = \frac{kQ}{r} \tag{P63-5}$$

To have the two potentials add to zero we will have to be closer to the smaller of the two charges; in this case that is the negative charge. With only that restriction there are two places we could find the total potential to be zero. The first would be between the two charges but closer to the negative one. The second would be to the right of the negative charge.

[‡‡] We are taking the potential at large distances to be zero as well.

Consider the first location. The distance from the positive charge will be x and the distance from the negative charge will be L-x. This last is because L > x. Now use (P63-5) twice to write the total potential at x.

$$V(x) = \frac{kQ}{x} + \frac{kq}{L-x} \tag{P63-6}$$

If V is zero then (P63-6) can be written as

$$-Q \cdot (L-x) = q \cdot x$$

$$or$$

$$x = L \cdot \left(\frac{Q}{Q-q} \right)$$

$$x = 1.6cm \cdot \left(\frac{3.4\mu C}{3.4\mu C + 2.6\mu C} \right)$$

$$x = 0.91cm$$

For the second location the distance from the positive charge is still x but he distance from the negative charge is now x – L because x > L. The replacement for (P63-6) is

$$V(x) = \frac{kQ}{x} + \frac{kq}{x-L} \tag{P63-7}$$

and we proceed as before.

$$-Q \cdot (x-L) = q \cdot x$$

$$or$$

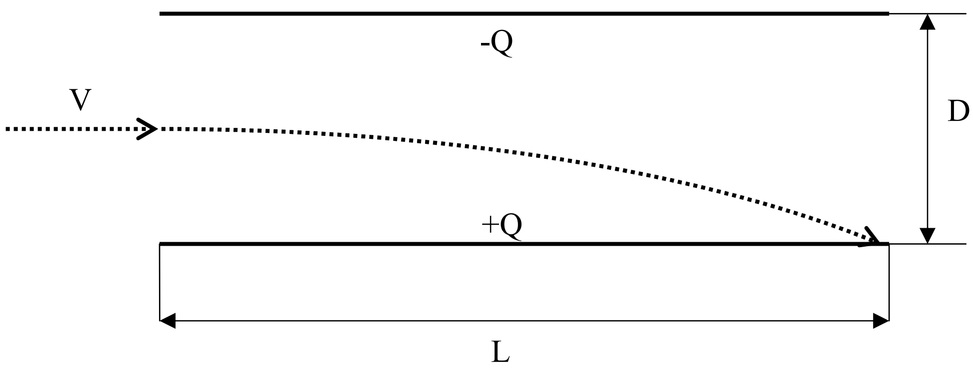
$$x = L \cdot \left(\frac{Q}{Q+q} \right)$$

$$x = 1.6cm \cdot \left(\frac{3.4\mu C}{3.4\mu C - 2.6\mu C} \right)$$

$$x = 6.80cm$$

Sometimes problems just have different cases that must be considered separately. It does make the solution longer but generally it does not make it more difficult.

A1 A parallel plate capacitor has square plates that are L meters on a side and that are separated by D meters. A charge -Q is placed on the top plate and a charge +Q is placed on the bottom plate. As indicated below, a mass m is shot in from the side and just crashes into the edge on the lower plate on its way out of the capacitor. If the speed of the mass when it entered was V, what must the charge on the mass have been? Assume that the capacitor is in deep space. There is no gravitational field present in this problem. Your answer will be given in terms of L, D, Q, m, and V.



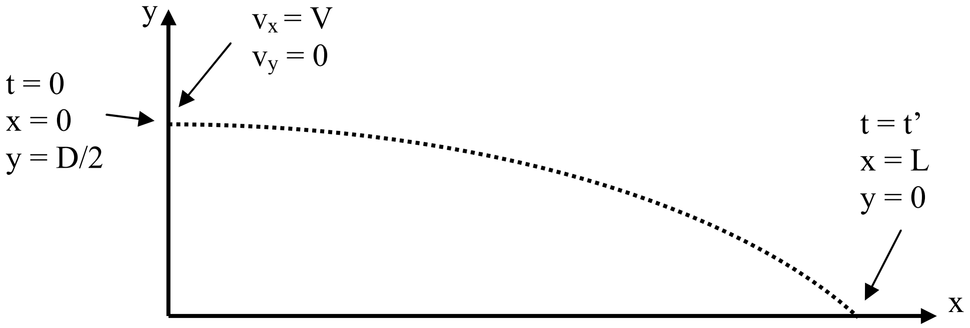
Many people find this a difficult problem to conceptualize. So, I will present an analogy and work through that. Then we can go back and see how this problem is very nearly identical.

If this were not a problem involving a charge and an electric field but instead it had been a rock chucked off a cliff on the surface of some planet with an acceleration of gravity g'[§§], how would you proceed? Take a few minutes to think about that.

We would state the rock off the cliff problem something like the following. The rock is thrown horizontally from the top of a cliff that is D/2 high and it lands a distance L from the bottom of the cliff. Find g'. We would proceed as in all of the projectile problems in chapter 3.

Draw a picture, put on a coordinate system, and label interesting points.

[§§] As the charge in the original problem is varied the size of the force pushing on the object changes. Its mass remains constant so its acceleration changes. Thus I expect that finding the acceleration will be closely related to finding the charge.



Now write down the constant acceleration position functions. Why can we use them?[***] Why do we not need the velocity functions?[†††]

$$x(t) = x_0 + v_{x,0}t + \frac{a_x t^2}{2} \tag{A1-1}$$

and

$$y(t) = y_0 + v_{y,0}t + \frac{a_y t^2}{2} \tag{A1-2}$$

We need to note the values of the constants in (A1-1) and (A1-2).

- $x_0 = 0$
- $v_{x,0} = V$
- $a_x = 0$
- $y_0 = D/2$
- $v_{y,0} = 0$
- $a_y = g'$

Apply the restrictions from our diagram.

$$\begin{gathered} x(t') = L \\ y(t') = 0 \end{gathered} \tag{A1-3}$$

Rewrite (A1-1) and (A1-2) using this information.

[***] Because all of the forces present (the weight) are constant in magnitude and direction.

[†††] Because the trajectory is described in terms of positions only in this problem.

$$x(t') = x_0 + v_{x,0}t' + \frac{a_x (t')^2}{2}$$

(A1-4)

$$L = V \cdot t'$$

$$y(t') = y_0 + v_{y,0}t' + \frac{a_y (t')^2}{2}$$

(A1-5)

$$0 = \frac{D}{2} - \frac{g' \cdot (t')^2}{2}$$

Solve (A1-4) and (A1-5) for g'. From (A1-4) we have $t' = L/V$ which can then be inserted into (A1-5)

$$0 = \frac{D}{2} - \frac{g' \cdot (L/V)^2}{2}$$

(A1-6)

$$g' = \frac{D \cdot V^2}{L^2}$$

So now the analogous problem is solved. What is g'? It is the rate at which the rock accelerates. What is the rate at which the mass in our original problem accelerates? Newton says that it is force/mass. The force is Q·E. Suddenly this problem has been reduced to finding the strength of an electric field inside a parallel plate capacitor. You can look that information up in your book or find it in your lecture notes. $E = \varepsilon_0 \frac{Q}{A}$ Here the constant ε_0 is related to the Coulomb constant and can be found on the inside cover of your book.

$$\frac{Q \cdot E}{m} = acceleration = \frac{D \cdot V^2}{L^2}$$

$$E = \varepsilon_0 \frac{Q}{A}$$

$$\varepsilon_0 \frac{Q^2}{A \cdot m} = \frac{D \cdot V^2}{L^2}$$

$$Q = \sqrt{\frac{A \cdot m \cdot D \cdot V^2}{\varepsilon_0 L^2}}$$